Analytic Geometry. By MARIA M. ROBERTS and JULIA T. COLPITTS. New York, Wiley, 1918. 10 + 229 pp. Price $1.60.

The following extracts from the preface serve to indicate the general plan of the book: "This book is the result of several years of experience in teaching mathematics to students of engineering and science. . . . Emphasis has been placed on those portions of analytic geometry in which experience has shown the student of calculus to be most frequently deficient. In this connection, in particular, polar coördinates have received more than usual attention and transcendental and parametric equations considerable space. . . . The material is so arranged that the first ten chapters together with a portion of Chapter XIII include those subjects ordinarily offered to such freshman classes as cover in the first year the three subjects, college algebra, trigonometry and analytic geometry. The addition of Chapter XIV will round out a good course of five hours a week for a semester. The entire book should easily be covered in a three-hour course throughout a year."

Chapter I (17 pages) is entitled Cartesian Coördinates. After the usual introduction, the formulæ for distance between two points, the slope of a line, the point of division, and the area of a triangle, are introduced and applied to numerical examples. Some familiar theorems from plane geometry are introduced for proof by analytic methods.

Chapter II (29 pages) is devoted to Loci. Here algebraic equations only are considered. In deriving the equation of a locus emphasis is placed on the fact that the student must show "why the coördinates of all points off the locus fail to satisfy the equation." Plotting the locus of an equation is considered at length. Most of the examples involve conic sections, although a few higher plane curves are introduced. The outline for the discussion of an equation is very good, particularly the section on extent of the curve. Curves with asymptotes parallel to one of the coördinate axes are introduced. The chapter closes with an interesting section on loci through the intersections of two given loci.

Chapter III (26 pages) is devoted to the straight line. Here the proof for the normal form of the equation, which is made to depend on the intercept form, is not satisfactory since it does not hold when the line passes through the origin. Moreover there seems to be no good reason why p in this equation should not be considered as always positive, instead of positive above the x axis and negative below as suggested by the authors. In fact, in the later chapter on solid analytic geometry, the perpendicular distance from the origin to a plane is always considered positive.

The fourth Chapter (17 pages) introduces polar coördinates. The treatment here is excellent but the subject is one which offers serious difficulties to the student. Since polar coördinates are not used elsewhere in the book, save in the chapter on transformation of coördinates, Chapter IV might well come later, after the student has acquired facility in methods of analytic geometry.

Transformation of coördinates comes in the fifth chapter, very appropriately preceding the chapters on the various conic sections. The treatment here is

brief (10 pages) but adequate. Translation as well as rotation of axes are considered and equations are simplified by each of these transformations. Examples involving the removal of the xy term are limited, however, to cases where a rotation of 45° is called for. The chapter concludes with a section on transformation from rectangular to polar coördinates and vice versa.

Chapters VI (20 pages), VII (10 pages), VIII (12 pages) and IX (14 pages) take up the circle, parabola, ellipse and hyperbola, respectively. In the case of the circle it seems hardly necessary to call $(x - h)^2 + (y - k)^2 = r^2$ the "First standard equation of a circle" and $x^2 + y^2 = r^2$ the "Second standard equation." Similarly the four standard equations of the ellipse and of the hyperbola could well be reduced to two, the others being merely special cases.

The chapter on the parabola begins with a derivation of the rectangular equation of a conic section from the ratio definition. The ratio definition is used also in deriving the equations of the ellipse and of the hyperbola, the familiar properties involving respectively the sum and difference of the focal radii being proved as theorems, not assumed as definitions. This seems on the whole the best method of approach. The treatment here is brief, only the fundamental properties of the various conics being discussed.

Tangents and normals are considered in the tenth chapter of twelve pages. The slope of a tangent is derived by means of the disguised calculus method with h and k substituted for Δx and Δy. This method is applied in particular to the general equation of the second degree, the resulting rule being very convenient in numerical examples. The chapter concludes with a paragraph on the lengths of tangents, normals, subtangents and subnormals and one on the equation of the tangent when the slope is given.

The remaining chapters take up topics of a more advanced nature and are not intended, as the preface indicates, for freshman classes. Poles, polars, diameters and confocal conics are discussed in the eleventh chapter of thirteen pages. The notion of pole and polar is introduced by means of harmonic division. The other topics are treated in the usual way.

A brief chapter (the twelfth) of only six pages follows. This takes up the general equation of the second degree. Methods for the classification of conics are developed and examples involving conics through five given points solved.

Chapter XIII (20 pages) involves transcendental and parametric equations. The usual logarithmic and exponential curves are plotted as well as the graphs of trigonometric and inverse trigonometric functions. Under parametric equations, some of the more important curves considered are the cycloid, the epicycloid, the hypocycloid, the path of a projectile, the witch and the cissoid.

The concluding chapter, numbered fourteen, is one of the best of the book. The material here is well selected and well presented. In the brief space of thirty-three pages one finds practically all the material regarding solid analytic geometry necessary for a first course in the calculus. In addition to the standard proofs and examples concerning the line and plane, the chapter includes discussions of cylindrical surfaces, spherical surfaces, surfaces of revolution, the more

important forms of quadric surfaces and the equations of a space curve. The statement that the polar coördinates of a point in space are ρ, α, β and γ (ρ being the radius vector and α, β and γ the direction angles) is not in accordance with the usual notation for polar coördinates. Moreover the use of four coördinates for a point in some cases and three in others is confusing to the student, even though he is told that the four are not independent.

The problems in the book are almost entirely from the field of pure mathematics, few from engineering or other sources being used. The answers are given for most of the problems, either immediately after the statement of the problem or in a list of answers at the end of the book. Sets of miscellaneous examples inserted from time to time furnish an opportunity for review.

As already indicated in quotations from the preface, the book is conveniently arranged for use either in a course for a term, a semester or a year. The traditional over emphasis on conic sections is relieved by the excellent chapters on polar coördinates, transcendental equations and parametric equations.

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PROVIDENCE, R. I.,
March 1, 1919.

Lezioni sulla teoria dei gruppi continui finiti di transformazioni. By L. BIANCHI. Pisa, E. Spoerri, 1918. 8vo. 6 + 590 pp. Price 25 lire.

This volume was written for the students of the Italian universities and is a reproduction, with few changes, of the course by the same author which appeared in lithographed form about fifteen years ago. It constitutes the second introductory treatise on continuous groups in the Italian language, an earlier one having been published by G. Vivanti under the title *Teoria dei gruppi di transformazioni*, 1898; translated into French by A. Boulanger, 1904.

The interest of Italian mathematicians in the general field of group theory is reflected by the fact that they have now in their own language at least five books on this subject. Two of these relate to finite groups, viz., the translation into Italian of Netto's *Substitutionentheorie* by G. Battaglini, 1885, and the excellent introduction entitled *Lezioni sulla teoria dei gruppi di sostituzioni*, 1910, by the author of the work now under review. Their other books on group theory are devoted to the groups of analysis and geometry, and include the well-known *Teoria dei gruppi discontinui e delle funzioni automorfe* by G. Fubini, 1908, in addition to the two works mentioned in the preceding paragraph.

L. Bianchi is well known to American mathematicians as the author of good books relating to each of the three great fields of mathematics—algebra, analysis and geometry. His wide range of mathematical knowledge and his extensive experience as a writer have qualified him admirably for the production of a work having such varied contact as a treatise on continuous groups should exhibit. In the present work he has restricted himself largely to a lucid exposition of the theories contained in the first volume of the *Theorie der Transformationsgruppen* by Lie-Engel, 1888–1893.